

Quasinormal modes prefer supersymmetry ?

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One ambiguity in loop quantum gravity is the appearance of a free parameter which is called Immirzi parameter. Recently Dreyer has argued that this parameter may be fixed by considering the quasinormal mode spectrum of black holes, while at the price of changing the gauge group to $SO(3)$ rather than the original one $SU(2)$. Physically such a replacement is not quite natural or desirable. In this paper we study the relationship between the black hole entropy and the quasinormal mode spectrum in the loop quantization of $N = 1$ supergravity. We find that a single value of the Immirzi parameter agrees with the semiclassical expectations as well. But in this case the lowest supersymmetric representation dominates, fitting well with the result based on statistical consideration. This suggests that, so long as fermions are included in the theory, supersymmetry may be favored for the consistency of the low energy limit of loop quantum gravity.

I. INTRODUCTION

Loop quantum gravity (LQG) has in the past years been further developed and made great success in the study of the quantum theory of geometry (please see [1] for recent review). One remarkable result is that with the power of spin networks the area and volume spectra can be derived and characterized by discrete values[2]. Nevertheless, in loop quantum gravity there is an important parameter γ called Immirzi parameter unfixed[3], implying that given a two dimensional surface intersected by an edge with label J of a spin network, the area of this surface can be determined only up to a freely adjustable parameter,

$$A(J) = 8\pi\gamma l_p^2 \sqrt{J(J+1)}, \quad (1)$$

where l_p is the Plank length (taking $\sqrt{\hbar}$ in geometrical units) and J takes positive half-integer. Consequently it is also due to the Immirzi parameter that the famous factor $\frac{1}{4}$ in Bekenstein-Hawking entropy formula,

$$S_{BH} = \frac{A}{4l_p^2}, \quad (2)$$

could not be uniquely determined when applying quantum geometry to derive the statistical entropy of black holes[4]. This has been viewed as the main unsatisfactory point of this approach for some years.

Recently, motivated by Hod's work on the asymptotic quasinormal modes of black holes[5], Dreyer has proposed a novel way to fix the Immirzi parameter in loop quantum gravity[6]. The basic idea and the main results can be summed as follows. Having studied the asymptotic behavior of the quasinormal modes of a Schwarzschild black

hole with the mass M , Hod conjectured that the real part of the highly damped quasinormal mode frequencies ω_{QNM} asymptotically approaches to a fixed quantity [5]

$$\omega_{QNM} = \frac{\ln 3}{8\pi M}. \quad (3)$$

This conjecture has recently been confirmed analytically by Motl[7]. Hod also proposed that, if we assume the Bohr's correspondence principle is applicable to black holes, such an asymptotic frequency of the quasinormal modes should be consistent with the one of the quanta induced by the minimum change ΔA in the quantized area of the event horizon [5]. Going further, Dreyer made a striking observation that the Immirzi parameter may be fixed by applying this idea to loop quantum gravity. As a consequence, it is shown in [6] that

$$\gamma = \frac{\ln 3}{2\sqrt{2}\pi}. \quad (4)$$

However, at the same time it also forces

$$J_{min} = 1, \quad (5)$$

where J_{min} denotes the lowest possible spin for the representation of the gauge group. This is not the answer as we expect based on the statistical principle. As in the case of quantum general relativity, the gauge group is $SU(2)$ and its representation is labelled by positive half-integer, we expect the minimum value J_{min} should be $\frac{1}{2}$, rather than one. The above observation by Dreyer seemingly indicates that we have to either sacrifice $SU(2)$ as the gauge group and replace it with $SO(3)$, or find a mechanism to explain why the expected most important contribution from $J = 1/2$ edges is suppressed. The former option is not quite physically natural or desirable. Thus some attempts to save $SU(2)$ as the relevant gauge group in loop quantum gravity have been made (for instance see [8, 9]), but there additional physical considerations or assumptions are needed.

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In this paper we intend to present another quite simple but elegant scheme to fix the Immirzi parameter by considering the supersymmetric extension of loop quantum gravity, but more importantly, we find in this case J_{min} is forced to take the minimum value of the representation of gauge group $Osp(1|2)$ which is perfectly consistent with the most probable value as we expect based on the statistical consideration.

II. AREA SPECTRUM IN $N = 1$ SUPERGRAVITY AND BLACK HOLE ENTROPY

$N = 1$ supergravity in Ashtekar-Sen variables was originally given by Jacobson in [10] and its canonical quantization has been extensively studied in [11, 12]. In this section we only recall the crucial results that we need here; for more details on this subject please see [11, 12]. Similar to the case in quantum general relativity, a generalized notion of supersymmetric spin networks can be introduced to construct the Hilbert space of canonical supergravity, while a key different ingredient from the ordinary one is that the edges in spin networks now are labelled by the representations of the supergroup $Osp(1|2)$ rather than $SU(2)$. Such a replacement gives rise to a different expression for the area spectrum at quantum mechanical level. More explicitly, if a surface is intersected by an edge of the network carrying the label J , then its area is given by [12]

$$A(J) = 8\pi\tilde{\gamma}l_p^2\sqrt{J(J + \frac{1}{2})}, \quad (6)$$

where J still takes half-integer but labels $Osp(1|2)$ representation and $\tilde{\gamma}$ denotes the Immirzi parameter in supergravity. If we simply set $\gamma = \tilde{\gamma}$ and compare it with the area spectrum (1) in ordinary loop quantum gravity, we find that the discrepancy is so tiny that it can be ignored for large J , while for small values of J these two sorts of spectra in principle should be distinguishable and perhaps yield quite different physics. We will soon see it does so for the statistical entropy of black holes as discussed below.

The derivation of statistical entropy of black holes in loop quantum gravity has been thoroughly studied and we refer to [4, 13] for details. Here we simply argue that the strategy can be directly extended to consider the entropy of black holes (at least for Schwarzschild black holes) in the context of supergravity. The only thing that should be stressed is that the dimension of the Hilbert space associated to a puncture with spin J on the event horizon now is [12]

$$D(J) = 4J + 1. \quad (7)$$

Therefore, the area of its event horizon with N punctures may be obtained as

$$A = N8\pi\tilde{\gamma}l_p^2\sqrt{J_{min}(J_{min} + \frac{1}{2})}, \quad (8)$$

and the number of microscopic quantum states contributes the black hole an entropy with

$$S = N \ln(4J_{min} + 1). \quad (9)$$

where we have used the fact that given a black hole with fixed horizon, the most probable distribution of spins associated to the punctures is the configuration in which all the punctures are labelled by the minimum value of spin, denoted as J_{min} . Combining Eq.(8) and Eq.(9), we find the black hole entropy is proportional to the area of the event horizon, i.e.

$$S = \frac{\ln(4J_{min} + 1)}{8\pi\tilde{\gamma}l_p^2\sqrt{J_{min}(J_{min} + \frac{1}{2})}} A. \quad (10)$$

Furthermore, comparing this with Bekenstein-Hawking formula (2) yields the following relation between the Immirzi parameter and the minimum value of spin,

$$\frac{\ln(4J_{min} + 1)}{8\pi\tilde{\gamma}\sqrt{J_{min}(J_{min} + \frac{1}{2})}} = \frac{1}{4}. \quad (11)$$

Next we follow the strategy advocated in [6] to fix the Immirzi parameter by considering the asymptotic quasinormal modes of black holes, but present a more reasonable value of J_{min} .

III. FIXING THE IMMIRZI PARAMETER BY QUASINORMAL MODE SPECTRUM

It is argued in [5, 6] that according to Bohr's correspondence principle, the increase of the black hole mass should be ascribed to a quanta with the energy $\hbar\omega_{QNM}$ absorbed by the black hole, i.e.

$$\Delta M = \hbar\omega_{QNM}. \quad (12)$$

On the other hand, since

$$A = 16\pi M^2, \quad (13)$$

such an increase in the mass of the black hole induces a corresponding increase in the area of the event horizon

$$\Delta A = 32\pi M \Delta M = 4 \ln 3 l_p^2. \quad (14)$$

At quantum mechanical level, this kind of increase in the area of the horizon corresponds to the appearance of another edge labelled with J_{min} in the supersymmetric spin network, thus

$$\Delta A = A(J_{min}). \quad (15)$$

Plugging it into Eq.(14) we have

$$\tilde{\gamma} = \frac{\ln 3}{2\pi\sqrt{J_{min}(J_{min} + \frac{1}{2})}}. \quad (16)$$

Therefore, Eq.(16) together with Eq.(11) uniquely fixes the value of the Immirzi parameter to be

$$\tilde{\gamma} = \frac{\ln 3}{\sqrt{2\pi}}. \quad (17)$$

At the same time it also determines

$$J_{min} = \frac{1}{2}. \quad (18)$$

This is a remarkable result because it is consistent with our argument based on statistical principle, stating that the most probable distribution should be the set of punctures on the horizon uniformly labelled by the fundamental representation of supergroup $Osp(1|2)$ which takes the minimum positive value of J . Here we do not need any other extra assumption or condition. It only seems implying that the supersymmetry might be a favorable nature of spacetime.

IV. DISCUSSION

We conclude this paper by pointing out a very interesting relation between two area spectra respectively obtained in quantum general relativity and the simplest supergravity, namely Eq.(1) and Eq.(6). Note in quantum gravity, we do not have any reason to argue that those two Immirzi parameters should be related in any manner. Therefore these two spectra in principle have nothing to do with each other. One naive but natural choice is just setting two Immirzi parameters equal as we assumed in section two such that at each level labelled by the same J the spectrum has only a small shift after the supersymmetry is concerned. In particular, such a shift is vanishing as J approaches to infinity. However, this picture has greatly changed when the semi-classical limit of these two theories is concerned. The asymptotic quasinormal modes of black holes do provide us a strategy to fix the Immirzi parameter so that these two sorts of spectra are comparable. Quite strikingly, here we found our

previous intuition is not quite correct. As a matter of fact, two Immirzi parameters are not equal but

$$\tilde{\gamma} = 2\gamma. \quad (19)$$

Therefore, the area spectrum in canonical supergravity can be rewritten as

$$A^{sg}(J) = 8\pi\gamma l_p^2 \sqrt{2J(2J+1)} \equiv 8\pi\gamma l_p^2 \sqrt{J_{eff}(J_{eff}+1)}. \quad (20)$$

Now it's evident that the area spectrum is the same expression as the one in quantum general relativity except that J_{eff} here can only take integer! That is to say, all the area eigenvalues with half-integers in loop quantum gravity now are forbidden by the supersymmetry in $N = 1$ supergravity.

Corichi and Swain have suggested to suppress the contribution from $J = 1/2$ edges in loop quantum gravity by considering the coupling of fermions or Pauli exclusion principle respectively. From this point of view, we may also argue that we have presented a very elegant scheme to do so, but the reason is simply due to the supersymmetry. Interestingly enough, all these suggestions share one common point, namely the involvement of fermions. As a result, it's quite desirable to investigate their relations and show more insight into this subject.

So far, we have only focused on Schwarzschild-like black holes. We expect the similar consideration will be applicable to more general black holes. At the same time, investigating the supersymmetric extension of loop quantum gravity with $N > 1$ is also under progress.

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